3.8

б) Найти длину дуги $r = a \cdot \sin^3 \frac{\varphi}{3}$

$$L = \int_{\varphi_1}^{\varphi_2} \sqrt{(r(\varphi))^2 + (r'(\varphi))^2}\, d\varphi,$$

$$r'(\varphi) = \frac{3a}{3} \sin^2 \frac{\varphi}{3} \cdot \cos \frac{\varphi}{3} = a \sin^2 \frac{\varphi}{3} \cos \frac{\varphi}{3}.$$

$$L = \int_0^{2\pi} \sqrt{a^2 \sin^6 \frac{\varphi}{3} + a^2 \sin^4 \frac{\varphi}{3} \cos^2 \frac{\varphi}{3}}\, d\varphi =$$

$$= a \int_0^{2\pi} \sqrt{\sin^4 \frac{\varphi}{3} \left(\sin^2 \frac{\varphi}{3} + \cos^2 \frac{\varphi}{3}\right)}\, d\varphi =$$

$$= a \int_0^{2\pi} \sin^2 \frac{\varphi}{3}\, d\varphi = a \int_0^{2\pi} \frac{1 - \cos \frac{2\varphi}{3}}{2}\, d\varphi =$$

$$= \frac{a}{2} \cdot \int_0^{2\pi} \left(1 - \cos \frac{2\varphi}{3}\right) d\varphi =$$

$$= \frac{a}{2} \int_0^{2\pi} d\varphi - \frac{a}{2} \cdot \frac{3}{2} \int_0^{2\pi} \cos \frac{2\varphi}{3}\, d\frac{2\varphi}{3} =$$

$$= \frac{a}{2}\varphi - \frac{3a}{4} \sin \frac{2\varphi}{3} \Big|_0^{2\pi} =$$

$$= a\left(\pi - \frac{3}{4} \sin \frac{4\pi}{3} + \frac{3}{4} \underbrace{\sin 0}_{0}\right) =$$

$$= a\left(\pi + \frac{3}{4} \cdot \frac{\sqrt{3}}{2}\right) = a\left(\pi + \frac{3\sqrt{3}}{8}\right).$$

ответ!