2.5.

1) $\int_0^1 \frac{\sqrt{x}\,dx}{1+\sqrt{x}} = \Bigg/$ сделаем замену $\sqrt{x}=t$, $x=t^2$, $dx=2t\,dt$

x	0	1
t	0	1

$\Bigg/$

$$= \int_0^1 \frac{t\cdot 2t\,dt}{1+t} = 2\int_0^1 \frac{t^2+1-1}{t+1}dt =$$

$$= 2\int_0^1 \frac{t^2-1}{t+1}dt + 2\int_0^1 \frac{dt}{t+1} = 2\int_0^1 \frac{(t-1)(t+1)}{(t+1)}dt +$$

$$+ 2\int_0^1 \frac{dt}{t+1} = 2\int_0^1 (t+1)\,dt + 2\int_0^1 \frac{d(t+1)}{t+1} =$$

$$= 2\left(\frac{t^2}{2}-t\right) + 2\ln(t+1)\Big|_0^1 =$$

$$= t^2-2t+2\ln(t+1)\Big|_0^1 = 1-2+2\ln 2 - \underbrace{2\ln 1}_{0} =$$

$$= -1+2\ln 2 \approx 0{,}368$$

2) $\int_2^\infty \frac{dx}{x\ln^3 x} = \int_2^\infty \frac{d\ln x}{\ln^3 x}$ – несобств. интеграл

$$\lim_{b\to\infty} \int_2^b \frac{d\ln x}{\ln^3 x} = \lim_{b\to\infty}\left(-\frac{1}{2\ln^2 x}\Big|_2^b\right) =$$

$$= \lim_{b\to\infty}\left(\frac{1}{2\ln^2 2} - \underbrace{\frac{1}{2\ln^2 b}}_{0=\frac{1}{\infty}}\right) = \frac{1}{2\ln^2 2} = 1{,}041.$$

сходится