3) $$\int_0^\infty \frac{dx}{x^2+x+1} = \int_0^\infty \frac{dx}{x^2+2\cdot\frac{1}{2}x+\frac{1}{4}+\frac{3}{4}} =$$

$$= \int_0^\infty \frac{d\left(x+\frac{1}{2}\right)}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$ – несобст. интеграл.

$$\lim_{b\to\infty} \int_0^b \frac{d\left(x+\frac{1}{2}\right)}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} =$$

$$= \lim_{b\to\infty} \left(\frac{2}{\sqrt{3}} \operatorname{arctg} \frac{\left(x+\frac{1}{2}\right)\cdot 2}{\sqrt{3}} \Bigg|_0^b \right) =$$

$$= \lim_{b\to\infty} \left(\frac{2}{\sqrt{3}} \left(\operatorname{arctg} \frac{2b+1}{\sqrt{3}} - \operatorname{arctg} \frac{1}{\sqrt{3}} \right) \right) =$$

$$= \frac{2}{\sqrt{3}} \left(\frac{\pi}{2} - \frac{\pi}{6} \right) = \frac{4\pi}{6\sqrt{3}} = \frac{2\pi}{3\sqrt{3}} \approx 1{,}209$$

сходится!