101.

$$\int_2^3 \frac{xdx}{\sqrt{x-1}} \left| \begin{array}{l} \sqrt{x-1}=t \\ x=t^2-1 \\ dx=2tdt \end{array} \right|$$

$$\int \frac{xdx}{\sqrt{x-1}} = \int \frac{\left(t^2-1\right)\cdot 2tdt}{t} = 2\int \left(t^2-1\right)dt =$$

$$= \frac{2}{3}t^3 - 2t + C = \frac{2}{3}\left(x-1\right)^{\frac{3}{2}} - 2\sqrt{x-1} + C$$

$$\int_2^3 \frac{xdx}{\sqrt{x-1}} = \frac{2}{3}\left(x-1\right)^{\frac{3}{2}} - 2\sqrt{x-1}\Bigg|_2^3 =$$

$$= \frac{4\sqrt{2}}{3} - 2\sqrt{2} - \frac{2}{3} - 2 = -\frac{2\sqrt{2}}{3} - \frac{8}{3}$$

102.

$$\int_1^4 \frac{\sqrt{x}dx}{x+2} \left| \begin{array}{l} \sqrt{x}=t \\ x=t^2 \\ dx=2tdt \end{array} \right|$$

$$\int \frac{\sqrt{x}dx}{x+2} = \int \frac{t\cdot 2tdt}{t^2+2} = 2\int \frac{t^2+2-2dt}{t^2+2} =$$

$$= 2\int \frac{\left(t^2+2\right)dt}{t^2+2} - 2\int \frac{2dt}{t^2+2} = 2\int dt - 4\int \frac{dt}{t^2+2} =$$

$$= 2t - 4\,\frac{1}{\sqrt{2}}\,arctg\,\frac{t}{\sqrt{2}} + C = 2\sqrt{x} - 2\sqrt{2}arctg\,\frac{\sqrt{x}}{\sqrt{2}} + C$$

$$\int_1^4 \frac{\sqrt{x}dx}{x+2} = 2\sqrt{x} - 2\sqrt{2}arctg\,\frac{\sqrt{x}}{\sqrt{2}}\Bigg|_1^4 =$$

$$= 4 - 2\sqrt{2}arctg\sqrt{2} - 2 + 2\sqrt{2}arctg\,\frac{1}{\sqrt{2}} =$$

$$= 2 - 2\sqrt{2}arctg\sqrt{2} + 2\sqrt{2}arctg\,\frac{1}{\sqrt{2}}$$